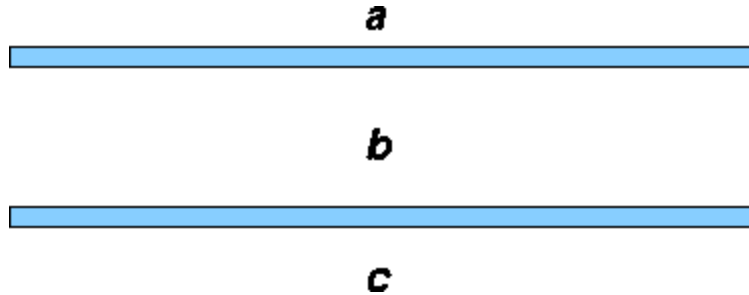# Solution Derivations for Capa #3

1) Consider two charged parallel planes of infinite extent as shown above. The magnitudes of the charge densities on the two planes are equal. (For each statement select T True, F False). WARNING! You have 4 tries only for this problem.

a

b

c

QUESTION:
A) If both plates are negatively charged, the electric field at a points towards the top of the page.
B) If both plates are positively charged, there is no electric field at b
C) If the plates are oppositely charged, there is no electric field at c

ANSWER:
A) False, the field points toward the bottom of the page.
B) True, the component of the electric field from each plane cancels the component from the other.
C) True, the electric field vector pointing away from the positive plane is cancelled by the field vector pointing toward the negative plane.

Remember to answer in True or False. For this problem, FTT

2) Consider a spherical CONDUCTING shell with NO NET CHARGE, with a point charge, $+Q$, placed at its center. (For each statement select T True, F False). WARNING! You have 8 tries only for this problem.

QUESTION:
A) The electric field at e is zero.
B) The electric field at a is zero.
C) The inner surface of the shell carries a charge $-Q$.
D) The electric field at c is zero.

ANSWER:
A) False, there is an enclosed charge, namely the point charge at the center. The overall shell has no *net* charge, however.
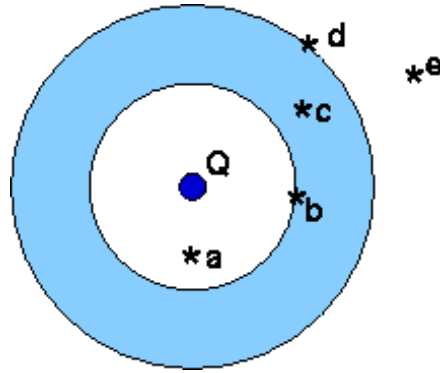
B) False, it encloses the point charge.

C) True. Charge must reside on a surface. The interior of the shell cannot have any electric field. Thus, a Gaussian surface through c must have no field. The only way for there to be no field through c is for the Gaussian surface to enclose no net charge.

D) True, see (C).

Again, you must enter True or False for this question. FFTT for this example.

3) Consider a sphere of radius $R = 8.5$ m where a charge $Q = 4.0$ μC is uniformly distributed through the volume of the sphere. What is the magnitude of the electric field at a point half way between the center of the sphere and the surface?

$R =$ Given
$Q =$ Given

The volume charge density of this sphere is

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3}$$

This also means that $Q = \rho V$. A point halfway from the center to the surface would have radius

$$r = \frac{R}{2}.$$

To find the volume this smaller sphere has,

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \left(\frac{R}{2}\right)^3 = \frac{4}{3}\pi \frac{R^3}{8} = \frac{1}{6}\pi R^3.$$

The charge enclosed will be proportional to the amount of volume the smaller sphere has compared to the larger sphere. Thus,

$$q_{enclosed} = \rho \left(\frac{1}{6}\pi R^3\right) = \frac{Q}{\frac{4}{3}\pi R^3}\left(\frac{1}{6}\pi R^3\right) = \frac{1}{8}Q.$$

2

Now, Gauss's law will tell us the flux through this surface which we can then relate to the enclosed charge.

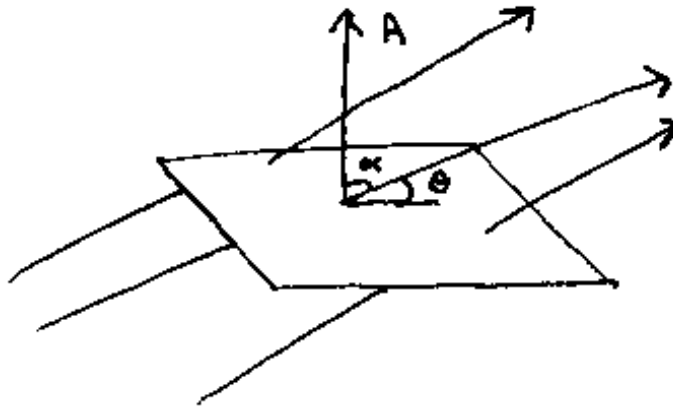$$\oint E \cdot dA = \oint E \cos\theta \, dA = E \oint dA = 4\pi r^2 E = 4\pi \left(\frac{R}{2}\right)^2 E = \pi R^2 E.$$

Now,

$$\oint E \cdot dA = \frac{q_{enclosed}}{\epsilon_0}.$$

So,

$$\pi R^2 E = \frac{1}{8}\frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{8\pi R^2 \epsilon_0} = \frac{kQ}{2R^2}$$

4) An electric flux of 159 $N \cdot m^2/C$ passes through a flat horizontal surface that has an area of 0.82 m^2. The flux is due to a uniform electric field. What is the magnitude of the electric field if the field points 15° above the horizontal?
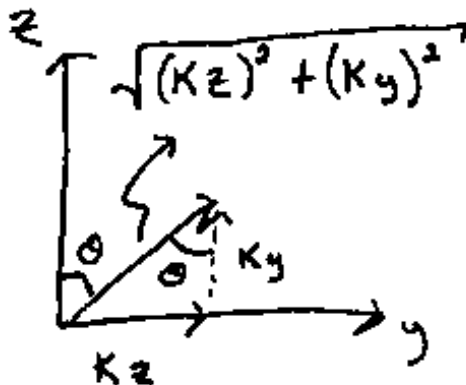

ϕ = Given
A = Given
θ = Given

The electric flux, you will remember, is also given by $E \cdot A$. This dot product is also equal to $EA\cos\theta$. However, θ in this case is the angle between the surface normal vector and the electric field vector. The angle you are given is the angle between the electric field and the surface. So, the angle between the normal vector and the field vector is $\alpha = (90 - \theta)$.

$$\phi = EA\cos\alpha$$

$$E = \frac{\phi}{A\cos\alpha}$$

3

5) A cubic cardboard box of side $a = 0.340\ m$ is placed so that its edges are parallel to the coordinate axes, as shown in the figure. There is NO net electric charge inside the box, but the space in and around the box is filled with a nonuniform electric field of the following form: $\vec{\mathbf{E}}\,(x, y, z) = Kz\,\mathbf{j} + Ky\,\mathbf{k}$, where $K = 3.40\ N/C \cdot m$ is a constant.

What is the electric flux through the top face of the box? (The top face of the box is the face where $z = a$. Remember that we define positive flux pointing out of the box.)

$a = $ Given
$\mathbf{E} = Kz\,\mathbf{j} + Ky\,\mathbf{k}$
$K = $ Given

In this case, we have to use a double integral to evaluate $\mathbf{E}$ over the entire surface. But, z is a constant across the top face. Thus, we don't have to worry about it in the integral. Since $\mathbf{E}$ also does not depend on x, we can think of the top face of the box simply as a square in the xy-plane. The equation we have is then

$$\phi = \int E \cdot dA = \int_{x=0}^{a} \int_{y=0}^{a} E \cos\theta \, dy \, dx$$

The E is the magnitude of the vector $\mathbf{E}$. To figure out what the magnitude of $\mathbf{E}$ and $\cos\theta$ really are, we have to think about the vector equation given for $\mathbf{E}$. This says that at any given point (x, y, z), the vector points Kz units in the y direction and Ky units in the z direction. To find the resultant vector, we use the Pythagorean theorem. Thus,

$$E = \sqrt{(Kz)^2 + (Ky)^2}.$$

To find the angle θ, we can construct a triangle. From this triangle, we can read off $\cos\theta$. Again, θ is the angle between the normal vector and the electric field vector. In this case, the normal vector will be pointing straight up (in the $+z$ direction). Using alternate interior angles, we can see that

$$\cos\theta = \frac{Ky}{\sqrt{(Kz)^2 + (Ky)^2}}.$$

4

Plugging this back into the integral,

$$
\begin{aligned}
\phi &= \int_{x=0}^{a} \int_{y=0}^{a} \frac{Ky}{\sqrt{(Kz)^2 + (Ky)^2}} \sqrt{(Kz)^2 + (Ky)^2}\, dy\, dx \\
&= \int_{x=0}^{a} \int_{y=0}^{a} Ky\, dy\, dx \\
&= K \int_{y=0}^{a} \left(yx\, dy\big|_{x=0}^{a} \right) \\
&= Ka \int_{y=0}^{a} y\, dy \\
&= Ka \left(\frac{y^2}{2} \bigg|_{y=0}^{a} \right) \\
&= \frac{1}{2} Ka^3
\end{aligned}
$$

6) What is the total electric flux through the 5 other faces of the box? (Again, outward flux is positive.)

We can think of the box as a Gaussian surface. Since there is no net charge inside the box, there will be no net electric flux thorough the surface. Thus, if one face has electric flux given in (5), then the total electric flux by the other 5 sides must cancel this. Thus, the answer is simply

$$
\phi = -\frac{1}{2} Ka^3
$$

7) A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting spherical shell with inner radius c and outer radius d. The inner shell has a total charge $-2q$ and the outer shell has a total charge $+4q$.
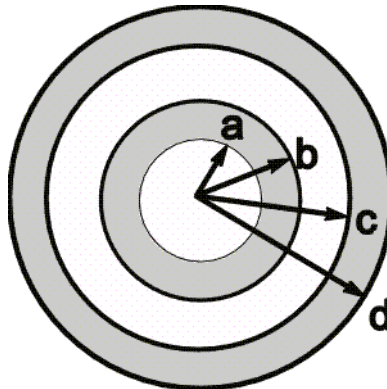
QUESTION:
A) The total charge on the inner surface of the small shell is zero.

B) The radial component of the electric field in the region $r < a$ is given by $-2q/\left(4\pi\epsilon_0 r^2\right)$.
C) The total charge on the outer surface of the large shell is $+2q$.
D) The radial component of the electric field in the region $c < r < d$ is given by $-2q/\left(4\pi\epsilon_0 r^2\right)$.
E) The radial component of the electric field in the region $r > d$ is given by $+2q/\left(4\pi\epsilon_0 r^2\right)$.
F) The total charge on the inner surface of the large shell is zero.
G) The total charge on the outer surface of the small shell is $-2q$.

ANSWER:
A) True, all the charges will move to the outside to get as far apart as possible.
B) False, the electric field is zero inside the shell.
C) True, the other $+2q$ will be on the inner surface, so a Gaussian surface in the outer shell has no enclosed charge.
D) False. There is no electric field inside of the shell.
E) True, shell acts like a point charge (see lecture notes, 24-6).
F) False, $+2q$ will be on the inside. See (C).
G) True, all the charges will move to the outside.

Remember to give the letters of the true answers, in this case, ACEG

8) A thin *infinite* nonconducting sheet with uniform surface charge density $\sigma = +8.0\ \mu C/m^2$ lies in the y-z plane. A charge $Q = +5.5\ \mu C$ is located on the x axis at a distance $x = 36\ cm$ from the sheet as shown.
    Find the magnitude of the electric field at a point P with coordinates $x = y = 36\ cm$, $z = 0$.

$\sigma =$ Given
$Q =$ Given
$x =$ Given

The electric field at any given point is the vector sum of fields from each charge present. The electric field from an infinite plane of charge (Lecture notes, 24-6) is

$$E_{plane} = \frac{\sigma}{2\epsilon_0}.$$

The field from a point charge is

$$E_{point} = \frac{q}{4\pi\epsilon_0 r^2}.$$

Remember to convert the distance to meters and the charges to Coulombs.

The magnitude is then

$$E = \sqrt{E_{plane}^2 + E_{point}^2}$$

9) A test charge of 2.90 μC is placed 6.30 cm away from a very large flat uniformly charged nonconducting surface. The force on the charge is 230 N. The charge is now moved to 3.50 cm away from the surface. What is the force on the test charge now?

$q = $ Given
$r_1 = $ Given
$F_1 = $ Given
$r_2 = $ Given
$F_2 = ?$

The electric field from an infinite plane is constant everywhere. Thus, $E = \frac{F}{q}$ is constant. Since the field is not changing and the test charge is not changing, the force cannot change either.

10) Three identical metal spheres, x, y, and z, are placed at the corners of an equilateral triangle, as shown below. Sphere z has a constant positive charge. Which vector could represent the force on z, when x is positive and y is negative, with the magnitude of the charge on y less than that on x? WARNING! You have 4 tries only for this problem.

Cannot generalize further. A simple problem, though. Draw the vectors from each charge and vector add them.

11) A cork ball of mass 7.30 g is placed between two very large horizontal planes of charge. The bottom plane has a uniform charge density of +0.94 $\mu C/m^2$, whereas the upper plane has a uniform charge density of −0.94 $\mu C/m^2$. The cork ball, which carries an unknown charge, is placed between the planes and is observed to float motionlessly. What is the charge on the ball?

$m = $ Given
$\sigma_1 = $ Given
$\sigma_2 = -\sigma_1 = $ Given

Since the charge density of the two planes are equal and opposite, the electric field is just twice that of one of the fields. Since the positive one is on the bottom, the field vector points straight up. The relationship between the electric field and the electric force is given by $F = qE$. For this problem, sum all the forces in the y-direction, set them equal to 0, and solve for the unknown. The electric field for an infinite (or very large) plane is

$$E = \frac{\sigma}{2\epsilon_0}.$$

Thus,

$$F = q\,(2E) = mg$$

since the field is twice that generated by one plane.

$$q\frac{2\sigma}{2\epsilon_0} = mg$$

$$q = \frac{\epsilon_0 mg}{\sigma}$$

12) The diagram below shows a dipole centered at the origin and along the x-axis. Determine an expression for the total electric field at a point A ($r_0 = 3.8L, 0$) in terms of q and L. Calculate the magnitude of the total electric field at A, when $q = 8.15 \times 10^{-7}\ C$ and $L = 44.2\ cm$.

$r_0 =$ Given
$L =$ Given
$q =$ Given

To solve this problem, we can calculate the electric field at each end of the dipole and then vector add the fields together. The electric field experienced by a point charge is

$$E = \frac{F}{q} = \frac{kq}{r^2}.$$

The field from the negative charge is

$$E_1 = \frac{-kq}{(L + r_0)^2}$$

The field from the positive charge is

$$E_2 = \frac{kq}{(r_0 - L)^2}$$

The total electric field is then

$$E_1 + E_2 = \frac{kq}{(r_0 - L)^2} - \frac{kq}{(L + r_0)^2}$$

13) Consider the situation as described in the diagram, and the following statements. If the statement is true, answer 'T', if it is false, answer 'F', and if the answer cannot be determined from the information provided, answer 'C'. For example, if 'A' and 'E' are true and there is not enough information to answer 'C' and the rest are false, then answer 'TFCFTF'.

QUESTION:
A) The direction of the net electric field at A is $\leftarrow$
B) The magnitude of the net electric field at point A is less than the magnitude of the net electric field at the origin.
C) The direction of the net electric field at the origin is $\leftarrow$

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D) The direction of the net electric field at a point on the negative y-axis is ↓
E) The direction of the net electric field at the point $(-r_0, 0)$ is →
F) The direction of the net electric field at a point on the positive y-axis is ↘

ANSWER:
A) False, it is to the right caused by the proximity to the positive charge.
B) True, both charges contribute to the electric field in the same direction.
C) True. The direction from (B) is to the left.
D) False, it will be to the left.
E) True, it is opposite that calculated in (12).
F) False, it will be to the left, as in part (D)

14) A point charge $+q$ is at the origin. A spherical Gaussian surface centered at the origin encloses $+q$. So does a cubical surface centered at the origin and with edges parallel to the axes. Select T-True, F-False, If the first is T and the rest F, enter TFFFF.

QUESTION:
A) The Electric Flux through the spherical surface is lesser than that through the cubical surface.
B) The area vector and the E-Field vector point in the same direction for all points on the cubical surface.
C) By symmetry, the E-Field at all points on the cubical surface is zero.
D) The flux through the spherical Gaussian Surface is independent of its radius.
E) Now q is moved from the origin but is sill within both the surfaces. The total flux through both surfaces is changed.

ANSWER:
A) False, the same amount of field lines exit both surfaces.
B) False, the E-Field vector will point radially from the enclosed point charge where the area vector will point parallel to the axes.
C) False. Same argument could be made for the sphere, but the E-Field is not zero there either.
D) True, since the field lines begin on the charge and travel to infinity, there is no dependence on the radius of the surface.
E) False, the flux is only the amount of field lines travelling through the surfaces.

15) The diagram below depicts a section of an infinitely long cylinder of radius R that carries a uniform (volume) charge density ρ. Develop an expression for the electric field anywhere inside the cylinder. To check your result, what is the magnitude of the electric field at $r = 7.70$ cm, where $R = 9.20$ cm and $\rho = 1.05$ $\mu C/m^3$? (Note: r is measured radially from the center axis of the cylinder.)

r = Given
R = Given

$\rho = $ Given

For this problem, it is important to choose the appropriate Gaussian surface. Important things to remember are that the Gaussian surface should reflect the original object's shape, the electric field must be constant, and $E \cdot dA$ must be constant. Since there is a cylinder of infinite length involved, we can choose our Gaussian surface to be a smaller cylinder of radius r. Gauss's law tells us

$$\oint E \cdot dA = \frac{q_{enclosed}}{\epsilon_0}.$$

To find the enclosed charge, we can use the given volume charge density, ρ. This tells us charge/volume. So, we have to multiply ρ by a volume to get the corresponding charge. The volume of the enclosed cylinder is $\pi r^2 l$. Thus, $q_{enclosed} = \rho \pi r^2 l$. Now we can solve this.

$$\oint E \cdot dA = \frac{\rho \pi r^2 l}{\epsilon_0}$$

Since the electric field is constant, we can pull it outside of the integral.

$$E \oint dA = \frac{\rho \pi r^2 l}{\epsilon_0}$$

The integral of dA is just the surface area of the Gaussian surface, in this case a cylinder.

$$
\begin{aligned}
E 2\pi r l &= \frac{\rho \pi r^2 l}{\epsilon_0} \\
E &= \frac{\rho \pi r^2 l}{2\pi r l \epsilon_0} \\
E &= \frac{\rho r}{2\epsilon_0}
\end{aligned}
$$

16) A thin plastic rod is bent so that it has a shape of a semicircle of radius $R = 8.20\ m$. An amount of charge $Q = 10.00\ C$ is distributed along the rod. What is the magnitude of the electric field at the center of the circle (point P)?

$R = $ Given
$Q = $ Given

This problem reverts back to using direct integration. The length of the semicircle is just πr (half of the circumference of a circle). Thus, the charge per unit length is

$$\lambda = \frac{Q}{\pi r}.$$

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Coulomb's law for an electric field is

$$E = \frac{kq}{r^2}$$

If we integrate along the length of the semicircle, each little piece of the semicircle will have a charge dQ and an associated field dE. However, we are only interested in the vertical component of the field. If we take the left side of the semicircle to be $\theta = 0$ and the right side to be $\theta = \pi$, then the vertical component is $\sin\theta$. We now have

$$dE = \frac{k\,dQ}{r^2}\sin\theta$$

We must now relate dQ to θ since that is the variable we will integrate with respect to. We know that λ is the charge per length. So if we multiply it by a small length corresponding to $d\theta$, then we should be able to finish. The formula for arc length is $r\theta$. Thus, $dQ = \lambda r\,d\theta$. But $\lambda = \frac{Q}{\pi r}$, so $dQ = \frac{Q\,d\theta}{\pi}$

$$
\begin{aligned}
dE &= \frac{kQ}{\pi r^2}\sin\theta\,d\theta \\
E &= \int_0^\pi \frac{kQ}{\pi r^2}\sin\theta\,d\theta \\
E &= \frac{kQ}{\pi r^2}\int_0^\pi \sin\theta\,d\theta \\
E &= \frac{kQ}{\pi r^2}\left(-\cos\theta\big|_0^\pi\right) = \frac{kQ}{\pi r^2}\left(1-(-1)\right) = 2\frac{kQ}{\pi r^2}
\end{aligned}
$$