Solution Derivations for Capa #5

1) A positive charge of $Q = 5.30$ μC is fixed in place. From a distance of $r_i = 4.40$ cm a particle of mass $m = 6.00$ g and charge $q = +3.70$ μC is fired with an initial speed of $v = 60.0$ m/s directly toward the fixed charge. What is the minimum distance r_f between the two charges?

$Q = $ Given
$r_i = $ Given
$m = $ Given
$q = $ Given
$v = $ Given
$r_f = ?$

This problem is easiest to solve with energy considerations.

$$
\begin{aligned}
KE_i + PE_i &= KE_f + PE_f \\
\frac{1}{2}mv^2 + \frac{kqQ}{r_i} &= 0 + \frac{kqQ}{r_f} \\
\frac{mv^2}{2kqQ} + \frac{1}{r_i} &= \frac{1}{r_f} \\
r_f &= \frac{1}{\frac{mv^2}{2kqQ} + \frac{1}{r_i}}
\end{aligned}
$$

2) The electric potential along the x-axis (in *kilo Volts*) is plotted versus the value of x, (in *meters*). Evaluate the x-component of the electrical force on a charge of 4.50 μC located on the x-axis at $x = 0.4$ m.

$q = $ Given

Once again, you must hit this problem right on the money. Get out the laser timers and calculate the exact distance between the points. Well, you need to be pretty accurate anyway. Remember that the electric field is

$$E = -\frac{dV}{dl}$$

and that force is

$$F = qE.$$

3) Four charges are arranged at the corners of a square as shown in the figure. Q_1 is at $(-L/2, L/2)$, Q_2 is at $(L/2, L/2)$, point a is at the origin and $Q_1 = Q_2 = -Q_3 = -Q_4$. Which of the following statements correctly describe the electric field $\vec{E}$ and the true potential at points a, b, and c? Enter the letters of true statements in alphabetical order; e.g. "ACF"

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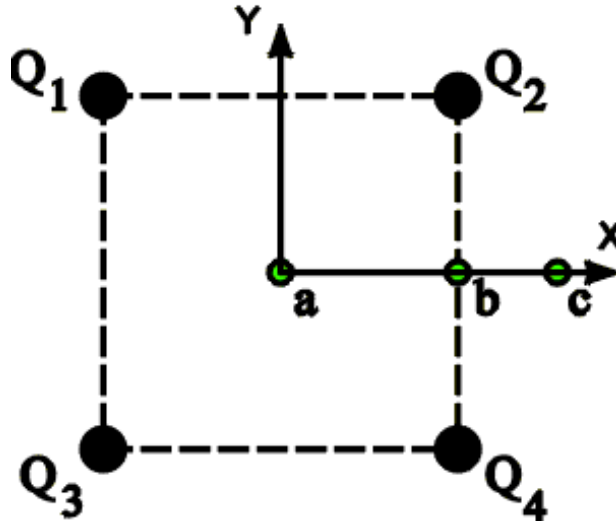
QUESTION:
A) The magnitude of the E field at b is greater than at c.
B) The E field at b points towards a.
C) The E field at c is perpendicular to the x axis.
D) The potential at a is zero.
E) a, b, and c each lie on a different equipotential surface.
F) The potential at b is greater (more positive) than the potential at a.
G) The E field at b is zero.

ANSWER:
A) True, b is closer to Q_2 and Q_4 than is c, so the electric field is strong.
B) False, the electric field points straight down, toward the negative charges.
C) True, see (B).
D) True, a is equidistant from all points. Potential is a scalar quantity and it sums to zero.
E) False, they are on the same equipotential surface (with the charges symmetrically distributed).
F) False, see (E).
G) False, see (A).

4) A capacitor consists of two closely spaced metal conductors of large area, separated by a thin insulating foil. It has an electrical capacity of 3200 μF and is charged to a potential difference of 63.0 V. Calculate the amount of energy stored in the capacitor.

C = Given
V = Given

Remember that energy is
$$U = \frac{1}{2}CV^2$$

Always remember to account for the units on C as they will probably not be SI.

5) Calculate the charge on this capacitor when the electrical energy stored in the capacitor is 12.11 J.

U = Given
Q = ?
Energy can also be expressed as

$$U = \frac{Q^2}{2C}$$
$$Q = \sqrt{2CU}$$

6) If the two plates of the capacitor have their separation increased by a factor of 4 while the charge on the plates remains constant, by what factor is the energy stored in the capacitor increased?

If the separation of capacitors is increased by x, then (from lecture notes 26-4),

$$U = \frac{1}{2}CV^2$$
$$= \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2$$
$$= \frac{1}{2}\varepsilon_0 E^2 (Ad)$$

which increases linearly by the same factor x.

7) Which statements are true for two oppositely charged, isolated parallel plates: (C = capacitance, U = stored energy, $+Q$ and $-Q$ = charge on the plates). Note: Isolated plates can not lose their charge. (Give ALL correct answers in alphabetical order, i.e., B, AC, BCD...)

QUESTION:
A) When the distance is doubled, C increases.
B) When the distance is halved, Q stays the same.
C) Inserting a dielectric increases Q.
D) Inserting a dielectric decreases U.
E) Increasing the distance increases the Electric field.
F) When the distance is doubled, U increases.
G) Inserting a dielectric increases C.

ANSWER:
A) False, $C = \frac{Q}{V} = \frac{Q}{Ed}$ which shows that capacitance decreases with an increase in distance.
B) True, if the plates are separated, there is no way for Q to change.

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C) False, see (B).

D) True, a dielectric will decrease the electric field strength. As shown in (6), U is directly proportional to E.

E) False, for a non infinite object, the electric field decreases as you move farther from it.

F) True, see (6).

G) True, inserting a dielectric will decrease the field strength. In (A), it was shown that capacitance is inversely proportional to the electric field. Decrease the field, increase the capacitance.

8) Three capacitors having capacitance $C_1 = 1.50 \ \mu F$, $C_2 = 9.50 \ \mu F$ and $C_3 = 17.0 \ \mu F$ are connected to a 28.0 V battery as shown. Calculate the charge on C_3.

Find the equivalent capacitance of the multiple capacitors. Capacitors in parallel are added while those in series add reciprocally. Then, from the expression

$$C = \frac{Q}{V},$$
$$Q = CV.$$

With the calculated capacitance and the given voltage, calculate the charge. Watch the units on the capacitance again.

9) A parallel plate capacitor with plate separation d is connected to a battery. The capacitor is fully charged to Q *Coulombs* and a voltage of V *Volts*. (C is the capacitance and U is the stored energy.) Give all correct answers concerning a parallel-plate capacitor charged by a battery (e.g. B, AC, CDF).

QUESTION:

A) With the capacitor connected to the battery, increasing d increases U.

B) With the capacitor connected to the battery, increasing d decreases Q.

C) After being disconnected from the battery, inserting a dielectric with $\kappa > 1$ will increase C.

D) With the capacitor connected to the battery, inserting a dielectric with $\kappa > 1$ will increase U.

E) After being disconnected from the battery, decreasing d increases C.

F) With the capacitor connected to the battery, inserting a dielectric with $\kappa > 1$ will increase Q.

ANSWER:

A) False, as shown in (7A), capacitance is proportional to the charge. The voltage is constant since the battery is attached and capacitance decreases with distance, so the charge must drop. Thus the energy, $U = \frac{1}{2}QV$ decreases.

B) True, see (A).

C) True, the charges are isolated, so Q is fixed. The electric field decreases in the presence of a capacitor. Thus, from (7A), the capacitance will increase.

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D) True, since the voltage is constant, increasing the capacitance with the dielectric will increase U by $U = \frac{1}{2}CV^2$.
E) True, see (7A).
F) True, since the voltage is constant and capacitance increases, the charge must increase.

10) An air-filled parallel plate capacitor with a plate spacing of 1.90 cm has a capacitance of 5.10 μF. The plate spacing is now doubled and a dielectric is inserted, completely filling the space between the plates. As a result the capacitance becomes 12.9 μF. Calculate the dielectric constant of the inserted material.

$d = $ Given
$C_i = $ Given
$C_f = $ Given

$$C_i = \frac{\varepsilon_0 A}{d}$$
$$C_f = \frac{\varepsilon_0 A}{2d}$$
$$2C_f = \frac{\varepsilon_0 A}{d} = C_i$$
$$\kappa = \frac{2C_f}{C_i}$$

This is so because this ratio will tell you how much the capacitance increased as a result of the distance change and the dielectric being inserted.

11) The large figure shows the equipotential surfaces for a long corner-shaped conductor and a long round conductor, both in a square metal enclosure. The enclosure is at ground potential (0 volts), the corner is at +4,000 volts, and the cylinder is at -4,000 volts. The intermediate voltage contours are 1000 $volts$ apart. The labels are in kV.

QUESTION:
A) The force on a (−) charge at 'c' points west.
B) The electric field E at 'k' points south.
C) The force on a (+) charge at 'e' points east.
D) The electric field E at 'f' is zero.
E) The electric field E at 'j' points east.
F) The potential energy of a charge at 'f' is zero.

ANSWER:
A) True, the electric field points at right angles to the potential lines. The (−) charge would be attracted to the positive charge, thus the force would be west.
B) True, a positive test charge would be repelled at k, along the electric field

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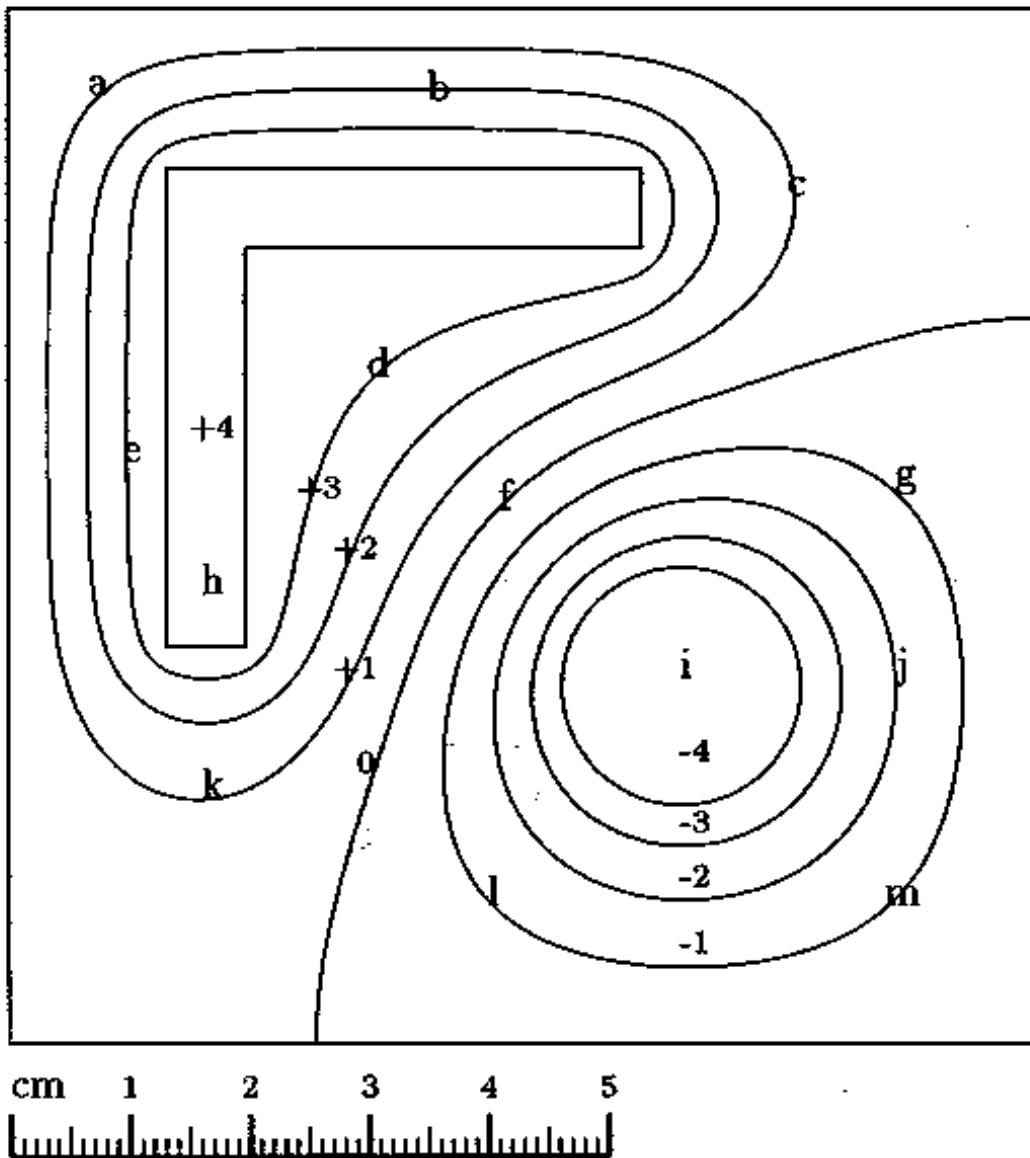
pointing downward.

C) False, the positive test charge wants to move away from the high positive charge already there, it will move west.

D) False, the voltage is still changing across 'f' even though it is instantaneously zero.

E) False, a positive test charge will be attracted to the negative charge, thus it will point west.

F) True... unsure exactly why...

12) Calculate the work required to move a charge of -0.55×10^{-12} C from 'm' to 'e'.

$q =$ Given

Work is defined as

$$W = qEl$$

But El is equal to the change in voltage, ΔV, between the two points. Thus, $W = q\Delta V$. Watch the sign.

13) Calculate the magnitude of the electric field at 'f'.

The electric field is defined to be

$$\begin{aligned} E &= -\frac{dV}{dl} \\ &\approx -\frac{\Delta V}{\Delta l} \end{aligned}$$

Thus, calculate the change in voltage across the point (the difference between the field line before and after it). Then, measure the distance, perpendicularly, from those two potential lines. Calculate the quotient and give the magnitude.