Solution Derivations for Capa #8

1) A mass spectrometer applies a voltage of $2.00\ kV$ to accelerate a singly charged ion $(+e)$. A $0.400\ T$ field then bends the ion into a circular path of radius $0.305\ m$. What is the mass of the ion?

$V =$ Given
$q =$ Given
$B =$ Given
$r =$ Given

A mass spectrometer will cause the particle to follow a circular pattern. Thus, we know that

$$F = ma = m\frac{v^2}{r} = qvB$$

where the last step follows as a result of the magnetic field. To find the mass of the ion, we can solve for m.

$$m\frac{v^2}{r} = qvB$$
$$m = \frac{qvBr}{v^2} = \frac{qBr}{v}$$

Now the only problem is to find r. It is related to the kinetic energy gained by accelerating though the electric field. Since it is initially at rest, and exits the electric field before entering the mass spectrometer, the final kinetic energy will be the potential energy gained by traveling though the electric field. That is,

$$\frac{1}{2}mv^2 = qV$$
$$v = \sqrt{\frac{2qV}{m}}$$

Plugging v into the above equation and squaring both sides,

$$m = \frac{qBr}{\sqrt{\frac{2qV}{m}}}$$
$$m^2 = \frac{q^2B^2r^2}{\frac{2qV}{m}} = \frac{mqB^2r^2}{2V}$$
$$m = \frac{qB^2r^2}{2V}$$

2) A charged particle is moving perpendicularly to a magnetic field B. Fill in the blank indicating the direction for the quantity missing in the table. (If your three answers from top to bottom are +z, -x, -y, then enter: +z-x-y in the computer). Use the diagram on the upper left for the directions of the various axes.

Use the right hand rule. Remember that if the charge is negative, it will move in the direction opposite that given by the right hand rule.

3) The rectangular loop shown below is located in a uniform magnetic field of 0.50 T pointing in the positive x direction. What is the magnitude of the magnetic dipole moment of the loop? DATA: $h = 6.5\ cm$, $w = 8.0\ cm$, $\theta = 36.0°$, $I = 5.0\ A$.

$h = $ Given
$w = $ Given
$\theta = $ Given
$I = $ Given
$B = $ Given

The magnetic dipole moment is defined by

$$\mu = IA = Ihw$$

The direction is given by curling your fingers in the direction of I. It is important for the next problem.

4) Calculate the magnitude of the torque about the z-axis on the loop.

Torque is given by
$$\tau = \mu \times B = |\mu|\,|B|\,\sin\phi$$

Imagine looking at the problem from the top. In this case, B is pointing down ($+x$ direction), and μ points to the upper right. The total angle between the two vectors is $180 - \theta$ (if μ is pointing more into the page than out). You can see this because if the wire loop were on the y-axis, μ would be antiparallel to B. Since the loop is tilted θ away from the y-axis, so is μ. Thus, $\phi = 180 - \theta$.

5) A proton, that is accelerated from rest through a potential of 7.0 kV enters the velocity filter consisting of a parallel-plate capacitor and a magnetic field, shown below. The E-field between the parallel capacitor plates is $3.7 \times 10^5\ N/C$. What B-field is required so that the protons are not deflected? (Ignore relativistic effects for high velocities.)

$V = $ Given
$E = $ Given
$q, m = $ Given (proton or electron)
$B = ?$

We can find the force of the electric field and the force due to the magnetic field. They must be equal for the particle to travel unaffected.

$$\begin{aligned} F &= qE = qvB \\ B &= \frac{E}{v} \end{aligned}$$

From problem 1, we know that $v = \sqrt{\frac{2qV}{m}}$. Thus,

$$B = \frac{E}{\sqrt{\frac{2qV}{m}}}$$

6) A mass spectrometer, sketched below, is a device used to separate different ions. Such ions with a well-defined velocity v_0 enter through a slit into a region of uniform magnetic field B, where they follow a semicircular path until they strike the detector slit D. The distance between the entry and the detector slits is $d = 1.10\ m$.

Chlorine ions of mass 35 amu (1 amu equals $1.66 \times 10^{-27}\ kg$), carrying a charge of $+1e$, enter the spectrometer with initial speed of $v_0 = 7.50 \times 10^5\ m/s$. What value of B is required for these ions to hit the detector?

$m = $ Given
$q = $ Given
$v = $ Given
$B = ?$

$$F = \frac{mv^2}{r} = qvB$$
$$B = \frac{mv}{qr}$$

Note that the mass will be some constant times one amu.

7) A point charge moving in a magnetic field of 1.18 $Tesla$ experiences a force of $0.744 \times 10^{-11}\ N$. The velocity of the charge is perpendicular to the magnetic field.

In this problem, we use the points of the compass and 'into' and 'out of' to indicate directions with respect to the page. If the magnetic field points west and the force points out of the page, then give all possible correct answers for the charge Q. (Give all correct statements in alphabetical order: ACD, BE, etc.)

$B = $ Given
$F = $ Given

If the magnetic field points in one direction, the force perpendicular to it, the particle must move in the other perpendicular direction. That is, it cannot move parallel to one of the other forces. That being said, use the right hand rule to find the direction of the force for a positive particle moving in the direction said. If the direction of the force is opposite to that given, the charge must be negative.

8) The speed of the charge in the problem above is $0.410 \times 10^8\ m/s$. Calculate the magnitude of the charge.

$v = $ Given

$q = $?

Using the equation for magnetic force,

$$\begin{aligned} F &= qvB \\ q &= \frac{F}{vB} \end{aligned}$$

9) In the problem above, the path of the charge in the magnetic field is a circle. Assume that the charge is positive and has a mass of 1.673×10^{-27} kg, the mass of a proton. What is the radius of the circle?

$m = $ Given

$r = $?

Using the equation for magnetic force and the charge calculated in the previous problem

$$\begin{aligned} F &= qvB = m\frac{v^2}{r} \\ r &= \frac{mv^2}{qvB} = \frac{mv}{qB} \end{aligned}$$

10) In the problem above, what is the orbital frequency, of the charge in the previous problem?

The book gives the equation (and the derivation) for frequency on page 729-730 (eqn 29-5). Using previously calculated and given values,

$$f = \frac{qB}{2\pi m}$$

11) Five particles follow the paths shown in the figure as they pass through a magnetic field B, which is indicated by the dots in the figure (B out of the page). What can you conclude about the charge of each particle? (If your answers for charges of particles 1, 2, 3, 4, and 5, are, respectively, $+$, 0, $+$, $-$, 0, then enter +0+-0 into the computer.)

Use the right hand rule to find the direction of the force. If the particle moves in the direction of the force, it is a positively charged particle. If it moves opposite of the force, it is negatively charged. If it is unaffected by the force, then it is a neutral particle.

12) A 7 kg particle carrying a charge of 65 μC enters a uniform 4 T magnetic field at a speed of 63 m/s and with an angle of 63° with respect to the field lines, as shown in the figure. Answer the following questions: (Select T-True, F-False, If the first is T and the rest F, enter TFFFF).

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QUESTION:
A) The y-component of the particle's velocity is unchanged as it passes through the B-Field.
B) The Force on the particle is in the -z direction.
C) The particle's speed varies as it passes through the B-Field.
D) The work done by the field on the particle is zero as the force is normal to the displacement.
E) The particle follows a helical path.

ANSWER:
A) False. Only the direction parallel to the magnetic field will be unchanged. Another way to see this is that the force into the page will cause the particle to move in a circular direction around the x-axis. This must change the y-velocity (and the z-velocity).
B) True. Use the right hand rule.
C) False. The magnetic field cannot change the particles speed, only the direction of the velocity.
D) True. The force of the magnetic field is always perpendicular to the movement of the particle. Thus, the dot product involved in work equates to zero.
E) True. As described in (A) and page 732 (fig. 29-15) of the book, a particle moving in a magnetic field will travel a helical path.