Solution Derivations for Capa #12

1) An ideal step-down transformer has a primary coil of 300 turns and a secondary coil of 25 turns. It is plugged into an outlet with 120 V (AC) from which it draws a current of 0.79 A. What is the voltage in the secondary coil?

N_p = Given
N_s = Given
V_p = Given (rms), subscript meaning "primary"
I_p = Given (rms)

The given values for voltage and current are the rms values because you know it is plugged into an electrical outlet. From the ratio

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

we can find the voltage in the secondary coil (which is also an rms value).

$$V_s = V_p \frac{N_s}{N_p}$$

2) Calculate the rms current in the secondary coil.

To find the rms current in the secondary coil, use the ratio

$$\frac{I_p}{I_s} = \frac{N_s}{N_p} = \frac{V_s}{V_p}$$

or

$$I_s = I_p \frac{N_p}{N_s}$$

Since the primary current I_p was given as an rms quantity, the secondary current, I_s, is also an rms value.

3) Assuming that the transformer secondary is driving a resistive load, calculate the average power dissipated in the resistor.

Average power is calculated from the equation

$$\bar{P} = I_{rms}V_{rms}$$

Where the rms current and rms voltage were calculated above.

4) The figure below shows the time variation of the current through an electrical heater when it is plugged into a 120 V, 60 Hz outlet.

V_{rms} = Given
f = Given

The voltage is an rms quantity since you are told the device is plugged into an outlet. The frequency confirms that it is a household outlet. The relationship to rms voltage and peak voltage is

$$V_{peak} = \sqrt{2}V_{rms}$$

5) A power supply that provides a voltage of $49.0\sin(388t)$ V is connected across a resistor $R = 200$ Ω. Calculate the average power dissipated in the resistor.

$V(t)$ = Given
R = Given

The power dissipated through a resistor is

$$P = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

You are given an equation for the voltage. The peak voltage is simply the amplitude of the sinusoidal function. The rms value is then calculated by

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$

So,

$$P = \frac{V_{rms}^2}{R} = \frac{1}{R}\left(\frac{V_{peak}}{\sqrt{2}}\right)^2 = \frac{V_{peak}^2}{2R}$$

6) A source of electromagnetic waves radiates power uniformly in all directions at a single frequency. At a distance of 6.50 km from the source, a detector measures the intensity of the wave to be 20.0 $\mu W/m^2$. What is the peak value of the magnetic field at the detector due to the waves?

d = Given
S = Given

From the lecture notes (pg. 34-6), we have several equations relating S to the rms and peak values of the electric and magnetic field. Choosing one with the peak magnetic field and solving for this quantity,

$$S = \frac{B_p^2 c}{2\mu_0}$$

$$B_p = \sqrt{\frac{2\mu_0 S}{c}}$$

7) What is the average power radiated by the source?

The quantity S is defined to be a power over an area. Since the electromagnetic waves are emitted radially from the source, the total power will be spread out over the surface of a sphere. Thus, by finding the ratio of the detector's surface are to the sphere's surface area, it is possible to determine the amount of power originally present.

$$S = \frac{P_0}{4\pi R^2}$$

where R is the radius from the source emitting the waves. In #6, the detector was a distance d from the detector. Thus, the original power is

$$P_0 = 4\pi S d^2$$

8) An electromagnetic wave from a wire antenna travels (from the reader) toward the plane of the paper. At time $t = 0$ s it strikes the paper at normal incidence. At point O and $t = 0$, the magnetic field vector has its maximum value, 5.40×10^{-8} T pointing in the negative y direction. The frequency of this wave is 1.80×10^{16} Hz. What is the x-component of the associated electric field $\vec{E}$ at time $t = 0$ s? (Use the right-hand rule to determine the direction of $\vec{E}$, and hence the sign of the x component.)

$B_{peak} =$ Given
$f =$ Given

Since the electromagnetic wave is in phase, both peak at the same time. The relationship between the two is given by

$$B_{peak} = \frac{E_{peak}}{c}$$

Or, equivalently,

$$E_{peak} = cB_{peak}$$

Since the wave is traveling into the page, the intensity (or Poynting vector) is also into the page. Using the right hand rule (with $S = E \times B$), the direction of E_{peak} is determined.

9) What is the magnitude of the Poynting vector of the wave at time $t = 0$ s?

$t =$ Given

Since the electric field is always perpendicular to the magnetic field in an electromagnetic wave, the intensity equation reduces to

$$S = \frac{E \times B}{\mu_0} = \frac{EB\sin\theta}{\mu_0} = \frac{EB\sin(90°)}{\mu_0} = \frac{EB}{\mu_0}$$

At $t = 0$, the magnetic field is at its maximum and so is the electric field.

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10) What is the y component of the magnetic field at point O at time 0.278×10^{-16} s?

$t = $ Given

You must calculate the number of periods elapsed in the time given in order to solve this problem. The frequency f was given in #8. The period relates to the frequency by:

$$T = \frac{1}{f}$$

The number of periods that have passed by in time t is calculated by dividing by the time necessary for one period.

$$N = \frac{t}{T} = tf$$

Then, you must think about the graph of sine. From the lecture notes on page 34-5, we know that the magnetic field oscillates according to:

$$B = B_{peak} \sin(kx - \omega t)$$

From the graph in #8, we know this is in the y-direction. Knowing the original magnitude and direction of B_{peak}, it is easy to calculate what the graph of sine looks like after N periods have passed.

11) A linearly polarized electromagnetic wave has an average intensity of 360 W/m^2. This wave is directed towards two ideal polarizers (in real polarizers, transmission is also effected by reflection and absorption). Polarizer A is oriented with its transmission axis at an angle of $\theta_1 = 26.5°$ with the incident electric field. Polarizer B has its axis at an angle of $\theta_2 = 76.5°$ with the incident electric field, as in the figure below. What is the average intensity of the wave after it goes through polarizer A?

$S_0 = $ Given
$\theta_1 = $ Given
$\theta_2 = $ Given

The Law of Malus, stated on page 878 of the textbook and 34-8 of the lecture notes, says that the transmitted intensity is given by

$$S = S_0 \cos^2 \theta$$

After passing through polarizer A which has the polarizing axis an angle θ_1 from perpendicular, the intensity is

$$S_A = S_0 \cos^2 \theta_1$$

12) What is the average intensity of the wave after it goes through polarizer B?

After the light passes through the second filter, its intensity drops again. However, its initial intensity is that of the light after it passes through filter A. Also, the angle between that which the light is traveling and that which it must pass through is $\theta_2 - \theta_1$. This is because the first polarizing filter only let light through at that angle. Thus,

$$S_B = S_A \cos^2(\theta_2 - \theta_1) = S_0 \cos^2\theta_1 \cos^2(\theta_2 - \theta_1)$$

13) Suppose that the two polarizers A and B are interchanged. What would the average intensity be after passing through both polarizers?

If the filters are reversed, the intensity is not the same. The calculation is nearly the same, however. The intensity after passing through the first filter is now

$$S_B = S_0 \cos^2\theta_2$$

The angle between the pass axis of the two filters is the same regardless of their orientation. That is, it does not matter which way the light is oriented toward the other filter as it will be the same angle difference. Thus, the intensity after passing through the second filter is then

$$S_A = S_B \cos^2(\theta_2 - \theta_1) = S_0 \cos^2\theta_2 \cos^2(\theta_2 - \theta_1)$$

14) The average intensity of solar radiation incident on Earth is 1390 W/m^2. What is the total power radiated by the sun? DATA: The mean distance between Earth and the sun is $1.5 \times 10^{11} m$.

$S = $ Given
$R = $ Given

This is nearly the same question as #7. We know that

$$S = \frac{P_0}{4\pi R^2}$$

so

$$P_0 = 4\pi S R^2$$

15) The average intensity of light from an industrial laser is $\bar{S} = 1.23 \times 10^9 \ W/m^2$. What is the rms value of the electric field?

$S = $ Given

Using an equation from page 34-6 which has the rms value of the electric field, we can solve directly for the answer.

$$S = \frac{E_{rms}^2}{\mu_0 c}$$
$$E_{rms} = \sqrt{S\mu_0 c}$$

16) For the same laser, what is the rms value of the magnetic field?

Recalling that $B_{peak} = \frac{E_{peak}}{c}$, we can solve for the rms value of the magnetic field by dividing both sides by $\sqrt{2}$.

$$\frac{B_{peak}}{\sqrt{2}} = \frac{E_{peak}}{\sqrt{2}c}$$

thus,

$$B_{rms} = \frac{E_{rms}}{c}$$

17) Select ALL the correct statements (AC, ADEF, etc.)

QUESTION:
A) The sun's radiation is most intense in the infrared region.
B) Gamma rays can be produced through electrons oscillating in wires of electric circuits.
C) Visible light is often emitted when valence electrons change their state.
D) Blue light has a longer wavelength than red.
E) Ultraviolet radiation causes common sunburn.
F) A vertical automobile antenna is sensitive to electric fields polarized vertically.
G) X-rays can be produced in transitions involving inner electrons in an atom.

ANSWER:
A) False, since we can see the sun, the most intense radiation is in the visible spectrum.
B) False, gamma rays can not be produced this way. Nuclear state transitions will produce gamma rays. See the diagram in the lecture notes after page 34-4.
C) True, valence electrons are the outer electrons of an atom. When these change states, visible light is emitted. See the diagram in the lecture notes after page 34-4.
D) False, from an electromagnetic spectrum chart, red has a longer wavelength than blue.
E) True.
F) True, if it were not, the atoms in the antenna would not have very far to move in order to pick up the signal.
G) True. See the diagram in the lecture notes after page 34-4.

CAPA is looking for an answer in the form CEFG.