Solution Derivations for Capa #14

1) An image of the moon is focused onto a screen using a converging lens of focal length ($f = 34.8\ cm$). The diameter of the moon is $3.48 \times 10^6\ m$, and its mean distance from the earth is $3.85 \times 10^8\ m$. What is the diameter of the moon's image?

$f =$ Given
$h =$ Given (diameter of moon, height of object)
$l =$ Given (distance to object)

The magnification equation can be used here:

$$\frac{h'}{h} = -\frac{l'}{l}$$

We are looking for h', the height (diameter) of the image on the screen. Since the moon is so far away, we can treat the light rays as being parallel. Thus, the image distance l' is the focal length of the lens.

$$h' = -\frac{hf}{l}$$

2) A $0.54\ cm$ high object is placed $8.5\ cm$ in front of a diverging lens whose focal length is $-7.5\ cm$. What is the height of the image?

$h =$ Given
$l =$ Given
$f =$ Given

We can't use the magnification equation immediately because we don't know the height of the image or the distance to it. But, the lens equation also relates these quantities. Thus, we can start with it and solve for the first unknown, the image distance.

$$\begin{aligned} \frac{1}{l} + \frac{1}{l'} &= \frac{1}{f} \\ \frac{1}{l'} &= \frac{1}{f} - \frac{1}{l} \\ l' &= \frac{1}{\frac{1}{f} - \frac{1}{l}} \end{aligned}$$

Now, we can use the magnification equation.

$$\begin{aligned} \frac{h'}{h} &= -\frac{l'}{l} \\ h' &= -\frac{hl'}{l} \end{aligned}$$

$$
\begin{aligned}
&= -\frac{h}{l}\frac{1}{\frac{1}{f}-\frac{1}{l}} \\
&= -\frac{h}{l\left(\frac{1}{f}-\frac{1}{l}\right)}
\end{aligned}
$$

3) A magnifying glass uses a converging lens with a focal length of 15.5 cm. It produces a virtual and upright image that is 2.7 times larger than the object. How far is the object from the lens?

f = Given
M = Given
l = ?

The magnification equation in another form is

$$M = -\frac{l'}{l}$$

Thus,

$$l' = -lM$$

Now, using the lens equation, the object distance can be solved for

$$
\begin{aligned}
\frac{1}{f} &= \frac{1}{l} + \frac{1}{l'} = \frac{1}{l} - \frac{1}{lM} = \frac{M}{lM} - \frac{1}{lM} \\
\frac{1}{f} &= \frac{M-1}{lM} \\
lM &= (M-1)\,f \\
l &= \frac{(M-1)\,f}{M}
\end{aligned}
$$

4) What is the image distance? (Think carefully about whether the answer is positive or negative.)

The image distance formula was a necessary step for the final formula in #3. It is

$$l' = -lM$$

Since l was calculated in #3, simple plug this in. Note the signs, the algebra will take care of it.

5) In the 7 diagrams below, the solid arrow represents the object and the dashed arrow the image. The rectangle shows the position of an SINGLE OPTICAL ELEMENT. Match each diagram with the appropriate optical element. (If the first corresponds to B, and the next 6 to C, enter BCCCCCC.)

Use the thin lens Java applet.

6) Which of the following statements are correct? (All of the statements concern real objects. Give ALL correct answers, i.e., C, AE, DEF, etc.)

QUESTION:
A) When an object is placed between a concave mirror and its focal point, the image is virtual.
B) The image produced by a convex mirror is always closer to the mirror than it would be in a plane mirror for the same object distance.
C) A concave mirror always forms a virtual image of a real object.
D) A concave mirror always forms an enlarged real image of a real object.
E) The virtual image formed by a convex mirror is always enlarged.
F) A virtual image formed by a concave mirror is always enlarged.
G) A convex mirror never forms a real image of a real object.

ANSWER:
Use the thin lens Java applet.

7) An apple is placed 12.6 cm in front of a diverging lens with a focal length of magnitude 22.0 cm. What is the image distance i of the image of the apple through this lens?

l = Given
f = Magnitude given, value is negative by convention
l' = ?

The lens equation will work here.

$$\begin{aligned} \frac{1}{l} + \frac{1}{l'} &= \frac{1}{f} \\ l' &= \frac{1}{\frac{1}{f} - \frac{1}{l}} \end{aligned}$$

8) What is the magnification of the image of the apple?

The magnification is given by

$$M = \left| \frac{l'}{l} \right|$$

9) Which of the following statements are true? (If, 'C' and 'D' are true and the rest are false, answer 'FFTT'.) The image of the apple is... (Note: "in front of" means same side as the object; "behind" means the other side.)

QUESTION:
A) In front of the lens.
B) Virtual.
C) Upright.

D) Larger in size.

ANSWER:
Due to the potential variability of this question, only remarks can be made.
A) For a diverging lens, the image will be in front of the lens. For a converging lens, the image will behind the lens for a real image, in front of the lens for a virtual image.
B) A diverging lens will always produce a virtual image. A converging lens will produce a real image unless the object is within the focal length.
C) A diverging lens' image will always be upright. A converging lens' image will be inverted if it's real, upright if it's virtual.
D) A diverging lens' image will always be reduced. A converging lens' image will be reduced if the object is greater than 2f away, otherwise it will be enlarged. This results since the object and image are the same size at 2f.

10) **NOTE:** We suggest you use ray diagrams to qualitatively understand these questions. A candle 8.40 *cm* high is placed in front of a thin converging lens of focal length 32.0 *cm*. What is the image distance i when the object is placed 94.0 *cm* in front of the same lens?

h = Given
f = Given
l = Given
l' = ?

A ray diagram will help with these problems. But, use the lens equation to work out mathematically what will happen.

$$\begin{aligned} \frac{1}{l} + \frac{1}{l'} &= \frac{1}{f} \\ l' &= \frac{1}{\frac{1}{f} - \frac{1}{l}} \end{aligned}$$

11) What is the size of the image? (Note: an inverted image will have a 'negative' size.)

The magnification equation will work here.

$$\begin{aligned} \frac{h'}{h} &= -\frac{l'}{l} \\ h' &= -\frac{hl'}{l} \end{aligned}$$

12) Is the image real(R) or virtual(V); upright(U) or inverted(I); larger(L) or smaller(S) or unchanged(UC); in front of the lens(F) or behind the lens(B)? Answer these questions in the order they are posed. (for example, if the image is real, inverted, larger and behind the lens, then enter 'RILB'.)

This is where the ray diagram will help.

13) A 79 year old man has a near point of 107 cm. What power lens (in $diopters$) does the man need in order to be able to see an object clearly at the normal near point of 25 cm? Assume that the person will get contact lenses, that is neglect the two cm that glasses would be away from the eye. Recall that diopters is $1/f$, where f is the focal length in meters. (Enter answer (in units of diopters) without units.)

Near Point $= l' =$ Given
Normal Near Point $= l =$ Given

The measurement of diopters is simply the ratio

$$p = \frac{1}{f}$$

where f is the focal length of the lens. The object of correcting the vision is to take an object at the normal near point and produce the image where the person's near point is. Thus, they will be able to focus. To do this, the object distance, l, is the normal near point. The image will be produced at the person's near point, labelled l' for the image distance. Thus,

$$p = \frac{1}{f} = \frac{1}{l} + \frac{1}{l'}$$

14) A 187x astronomical telescope is adjusted for a relaxed eye when the two lenses are 1.49 m apart. What is the focal length of the eyepiece?

$m =$ Given
$d =$ Given (separation of lenses)

The angular magnification for telescopes has the following definition

$$m = \frac{f_o}{f_e}$$

where f_o is the objective lens focal length and f_e is the eyepiece focal length. Since we are trying to solve for the eyepiece focal length, it will be easier to solve this equation for f_o.

$$f_o = m f_e \tag{1}$$

Presumably in a telescope, the observer is looking at an object very far away. Thus, the object distance is effectively infinity (i.e. $l_o = \infty$). The first lens in a telescope will take the distant object and place the image at the focal point of the lens (since the rays from the object will be effectively parallel, so $l'_o = f_o$). This image at the focal point will be the object for the eyepiece. The object will be at the focal point of the eyepiece lens (i.e. $l_e = f_e$). This way, the rays will emerge parallel from the eyepiece (which will by themselves produce an image at infinity) but will be focused by the eye into a point. Thus, the focal length of the objective lens and the focal length of the eyepiece will be equal to the total separation between the lenses. So,

$$f_e + f_o = d$$

But from the above reasoning and with equation (1), this becomes

$$\begin{aligned} f_e + m f_e &= d \\ f_e (1 + m) &= d \\ f_e &= \frac{d}{(1 + m)} \end{aligned}$$

15) Two narrow slits are illuminated by a laser with a wavelength of 538 nm. The interference pattern on a a screen located $x = 5.20\ m$ away shows that the second-order bright fringe is located $y = 9.40\ cm$ away from the central bright fringe. Calculate the distance between the two slits.

$\lambda =$ Given
$x =$ Given
$y =$ Given
$m =$ Given

Since this fringe is located a distance x away from the slit and a distance y from the center, a triangle is formed. The angle that the ray producing this fringe makes with the central ray is then

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

From the book, the additional distance the second ray must travel before hitting the same point on the screen is $d \sin\theta$. When this distance is equal to an integer number of wavelengths, constructive interference results in a bright "fringe." The order of the fringe is the number of additional wavelengths the second ray travelled from the first. Zero order is the center fringe, first order is the next, etc. Thus,

$$\begin{aligned} d \sin\theta &= m\lambda \\ d &= \frac{m\lambda}{\sin\theta} \end{aligned}$$